22137208

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Tuesday 21 May 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 9]*

The Taylor series of $\sqrt{x}$ about $x=1$ is given by

$$a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + \dots$$

(a) Find the values of a_0, a_1, a_2 and a_3. *[6 marks]*

(b) Hence, or otherwise, find the value of $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}$. *[3 marks]*

2. *[Maximum mark: 15]*

Consider the differential equation $\dfrac{dy}{dx} + y\tan x = \cos^2 x$, given that $y=2$ when $x=0$.

(a) Use Euler's method with a step length of 0.1 to find an approximation to the value of y when $x=0.3$. *[5 marks]*

(b) (i) Show that the integrating factor for solving the differential equation is $\sec x$.

(ii) Hence solve the differential equation, giving your answer in the form $y = f(x)$. *[10 marks]*

3. *[Maximum mark: 11]*

Consider the infinite series $\sum\limits_{n=1}^{\infty}\dfrac{n^2}{2^n}x^n$.

(a) Find the radius of convergence. *[4 marks]*

(b) Find the interval of convergence. *[3 marks]*

(c) Given that $x=-0.1$, find the sum of the series correct to three significant figures. *[4 marks]*

4. *[Maximum mark: 11]*

(a) Express $\dfrac{1}{r(r+2)}$ in partial fractions. *[3 marks]*

(b) Let $S_n = \sum_{r=1}^{n} \dfrac{1}{r(r+2)}$.

(i) Show that $S_n = \dfrac{an^2 + bn}{4(n+1)(n+2)}$, where a and b are positive integers whose values should be determined.

(ii) Write down the value of $\lim_{n \to \infty} S_n$. *[8 marks]*

5. *[Maximum mark: 14]*

(a)

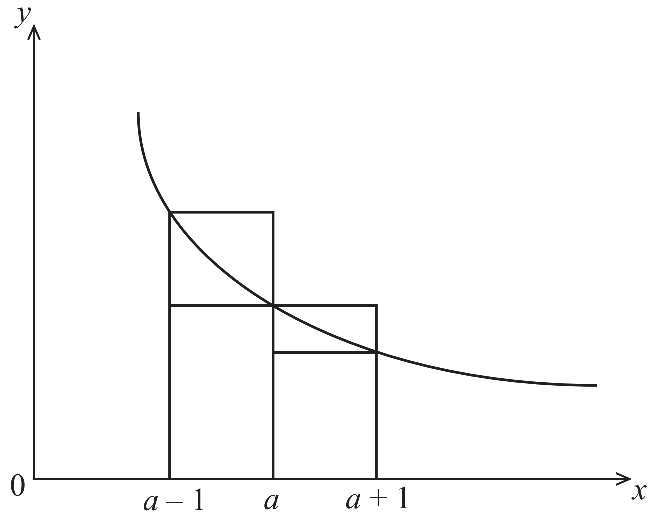

Figure 1

Figure 1 shows part of the graph of $y = \dfrac{1}{x}$ together with line segments parallel to the coordinate axes.

(i) By considering the areas of appropriate rectangles, show that

$$\frac{2a+1}{a(a+1)} < \ln\left(\frac{a+1}{a-1}\right) < \frac{2a-1}{a(a-1)}.$$

(ii) Hence find lower and upper bounds for $\ln(1.2)$. *[9 marks]*

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(Question 5 continued)

(b)

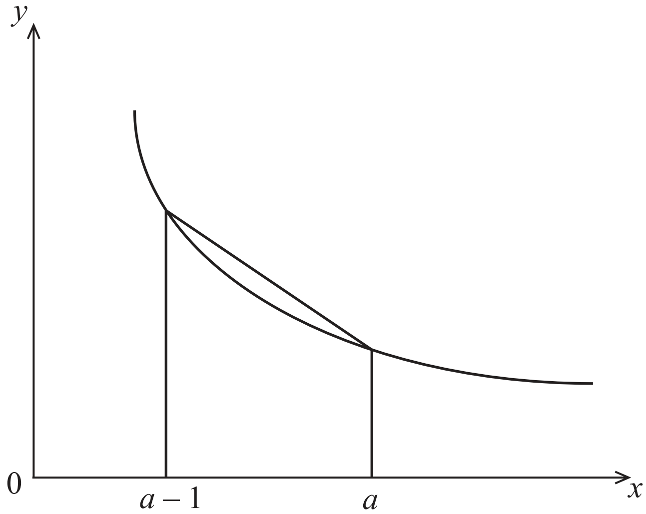

Figure 2

An improved upper bound can be found by considering Figure 2 which again shows part of the graph of $y = \frac{1}{x}$.

(i) By considering the areas of appropriate regions, show that

$$\ln\left(\frac{a}{a-1}\right) < \frac{2a-1}{2a(a-1)}.$$

(ii) Hence find an upper bound for $\ln(1.2)$. *[5 marks]*